Face-antimagic labelings

A labeling of type (1, 1, 1) assigns labels from the set \( \{1, 2, 3, \ldots, |V(G)| + |E(G)| + |F(G)|\} \) to the vertices, edges and faces of plane graph \( G \) in such a way that each vertex, edge and face receives exactly one label and each number is used exactly once as a label.

A labeling of type (1, 0, 0) is a bijection from the set \( \{1, 2, 3, \ldots, |V(G)| + |E(G)|\} \) to the vertices and edges of plane graph \( G \).

If we label only vertices (respectively edges, faces) we call such a labeling a vertex (respectively edge, face) labeling and also the labeling is said to be of type (1, 0, 0) (respectively type (0, 1, 0), type (0, 0, 1)).

The weight of a face under a labeling is the sum of labels (if present) carried by that face and the edges and vertices surrounding it.

A labeling of plane graph \( G \) is called \( d \)-antimagic if for every number \( s \) the set of \( s \)-sided face weights is \( W_s = \{a_s, a_s + d, a_s + 2d, \ldots, a_s + (f_s - 1)d\} \) for some integers \( a_s \) and \( d \), \( d \geq 0 \), where \( f_s \) is the number of \( s \)-sided faces.

We allow different sets \( W_s \) for different \( s \). If \( s \) is the same for each face, then there is just one arithmetic sequence comprising the set of face weights and we may speak of a graph being \( (a, d) \)-face antimagic. Many common types of plane graphs have "almost" all faces the same, for example, the prism which consists of all-but-two 4-sided faces; or the antiprism which consists of all-but-two 3-sided faces. Such graphs are easily modified so that they contain all the same faces and so that we can consider \( (a, d) \)-face antimagic labeling on them.

If \( d = 0 \) then Ko-Wei Lih called such labeling magic (face-magic).

If \( d = 1 \) then \( d \)-antimagic labeling is called consecutive.

Face-antimagic labelings are investigated in the following papers:

- Baˇca, M.- Bashir, F.: On super \( d \)-antimagic labelings of disjoint union of


