Magic, prime-magic and super-magic labelings

Graph G is *magic* (J.Sedláček, 1963) if and only if there exists a mapping f from $E(G)$ into the set of positive integers such that

(i) $f(e_i) \neq f(e_j)$ for all $e_i \neq e_j$; $e_i, e_j \in E(G)$,

(ii) $\sum_{e \in E(G)} \rho(v,e) f(e) = \lambda$ for all $v \in V(G)$,

where $\rho(v,e)$ is 1 when the vertex v and the edge e are incident and 0 in the opposite case.

The magic graphs in which each $f(e)$ is a prime number are called *prime-magic*.

We say that G is *super-magic* (B.M. Stewart, 1967) if there exists a magic labeling f such that the set $\{f(e) : e \in E(G)\}$ consists of consecutive integers.

The following papers deal with *magic* or *prime-magic* or *super-magic* labelings.

• Bača, M.: *On certain properties of magic graphs*, **Utilitas Math. 37** (1990), 259-264.

• Bača, M. - Holländer,I.: *Prime-magic labelings of* $K_{n,n}$, **J. Franklin Inst. 327** (1990), 923-926.

• Bača, M. - Holländer, I. - Ko-Wei Lih: *Two classes of super-magic quartic graphs*, **JCMCC 23** (1997), 113-120.

Vertex-magic total labelings

A one-to-one map λ from $V \cup E$ onto the integers $\{1, 2, \ldots, |V|+|E|\}$ is a *vertex-magic total labeling* if there is a constant k so that for every vertex x,

$$\lambda(x) + \sum \lambda(xy) = k$$

where the sum is over all vertices y adjacent to x.

The following papers study vertex-magic total labelings for generalized Petersen graphs and for several classes of convex polytopes.

• Sugeng, K.A. - Herawati, B.N. - Miller, M. - Bača, M.: *On magicness and antimagicness of the union of* 4*-regular circulant graphs*, **Australasian Journal of Combinatorics 50** (2011), 141-153.

• Miller, M. - Bača, M. - MacDougall, J.A.: *Vertex-magic total labeling of generalized Petersen graphs and convex polytopes*, **The Journal of Combinatorial Mathematics and Combinatorial Computing (JCMCC) 59** (2006), 89-99.

• Bača, M. - Miller, M. - Slamin: *Vertex-magic total labelings of generalized Petersen graphs*, **Intern. J.Computer Math. 79** (2002), 1259-1263.

• Bača, M., MacDougall, J.A. - Miller, M. - Slamin - Wallis, W.D.: *Survey of certain valuations of graphs*, **Discussiones Mathematicae Graph Theory 20** (2000), 219-229.

• Bača, M.: *Consecutive-magic labeling of generalized Petersen graphs*, **Utilitas Mathematica 58** (2000), 237-241.