Harmonious labelings

Harmonious graphs naturally arose in the study of modular version of error-correcting codes and channel assignment problems. Graham and Sloane in 1980 defined a (p, q)-graph G of order p and size q to be *harmonious* if there is an injective function $f : V(G) \to \mathbb{Z}_q$, where $\mathbb{Z}_q$ is the group of integers modulo q, such that the induced function $f^* : E(G) \to \mathbb{Z}_q$, defined by $f^*(xy) = f(x) + f(y)$ for each edge $xy \in E(G)$, is a bijection.

The function f is called a *harmonious labeling* and the image of f denoted by $Im(f)$ is called the corresponding set of vertex labels.
When G is a tree or in general for a graph G with $p = q + 1$, exactly one label may be used on two vertices.

Chang, Hsu and Rogers in 1981 define an injective labeling f of a graph G with q edges to be *strongly c-harmonious* if the vertex labels are from the set $\{0, 1, \ldots, q-1\}$ and the edge labels are from the set $\{f^*(xy) = f(x) + f(y) : xy \in E(G)\} = \{c, c+1, \ldots, c+q-1\}$. Grace in 1983 called such a labeling *sequential*. In the case of a tree, Grace allows the vertex labels to range from 0 up to q. Strongly 1-harmonious graph is called strongly harmonious.

By taking the edge labels of a sequentially labeled graph with q edges modulo q, we obviously obtain a harmoniously labeled graph. It is not known if there is a graph that can be harmoniously labeled but not sequentially labeled. More than 50 papers have been published on harmonious labelings.

In the next paper we study the existence of harmonious labelings for the certain families of graphs.

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