Homework #3

Problem # 5

Find a solution of the fractional integral equation:

$$Y(t) + \lambda\, {}_0D_t^{-1/2} Y(t) + = \frac{1}{\sqrt{\pi t}} \qquad (t > 0)$$

Problem # 6

Solve the initial value problem:

$$\frac{dv(t)}{dt} = g - \sqrt{b}\ {}_0^C D_t^{1/2} v(t)$$

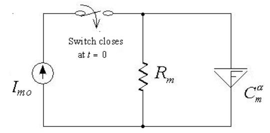
$$v(0) = v_0$$

Problem # 7

Solve the equation of the fractional order model of membrane charging

$$C_m^\alpha \frac{d^\alpha V_m(t)}{dt^\alpha} + \frac{1}{R_m} V_m(t) = I_{mo} u(t)$$

in terms of either R-L or Caputo derivatives under zero initial conditions.



Schematic diagram for a fractional charging model of membrane capacitance. The fractional capacitor element C_m^α is placed in parallel with the membrane resistance R_m.

Suggestion: denote

$$\tau^\alpha = R_m C_m^\alpha$$

Problem # 8

From:
Magin R. Fractional Calculus in Bioengineering.
Begell House Inc., Redding; 2006, p. 204,

Find a general solution of the following FDE:

$${}_0D_t^{2\alpha} y(t) + a_1\ {}_0D_t^\alpha y(t) + a_2\ y(t) = 0$$

$$(0 < \alpha < 1/2)$$

Problem # 10

Solve the following FDE for zero initial conditions:

$$\tau^\alpha\ {}_0D_t^\alpha V_F(t) + V_F(t) = V_0 \sin \omega t$$

Then consider $\alpha = 1/2$