Homework #1

1.1. Check if the functions

$$y_3(t) = t^{(\beta-1)n/m} E_{m/n,\beta}(t),$$

$$y_4(t) = t^{(\beta-1)n} E_{1/n,\beta}(t),$$

satisfy the following equations:

$$\left(\frac{m}{n} t^{1-\frac{n}{m}} \frac{d}{dt}\right)^m y_3(t) - y_3(t) = t^{(\beta-1)n/m} \sum_{k=1}^{n} \frac{t^{-k}}{\Gamma\left(\beta - \frac{m}{n}k\right)}$$

$$(m, n = 1, 2, 3, \ldots)$$

$$\frac{1}{n}\frac{dy_4(t)}{dt} - t^{n-1} y_4(t) = t^{\beta n-1} \sum_{k=1}^{n} \frac{t^{-k}}{\Gamma\left(\beta - \frac{k}{n}\right)},$$

$$(n = 1, 2, 3, \ldots)$$

1.2. Using the relationships

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

$$\sum_{\nu=-m}^{m} e^{i2\pi\nu k/(2m+1)} = \begin{cases} 2m+1, & \text{if } k \equiv 0 \pmod{2m+1} \\ 0, & \text{if } k \not\equiv 0 \pmod{2m+1} \end{cases}$$

demonstrate that

$$E_{\alpha,\beta}(z) = \frac{1}{2m+1} \sum_{\nu=-m}^{m} E_{\alpha/(2m+1),\beta}\left(z^{1/(2m+1)} e^{i2\pi\nu/(2m+1)}\right), \quad (m \geq 0)$$

1.3. Using term-by-term integration and

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}$$

demonstrate that

$$\int_0^z E_{\alpha,\beta}(\lambda t^{\alpha}) t^{\beta-1} dt = z^{\beta} E_{\alpha,\beta+1}(\lambda z^{\alpha}), \quad (\beta > 0)$$