

Fractional order models and fractional differential equations in science and engineering

Igor Podlubny
Technical University of Kosice
Slovak Republic

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6. Other Methods for the Solution of Fractional-order Equations

The Mellin Transform Method. Power Series
Method. Babenko's Symbolic Calculus Method.
Method of Orthogonal Polynomials.

7. Numerical Evaluation of Fractional Derivatives

Approximation of Fractional Derivatives. The "Short
Memory" Principle. Order of Approximation.
Computation of Coefficients. Higher-order
Approximations.

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1. Introduction and overview

2. Special Functions of the Fractional Calculus

Gamma Function. Mittag-Leffler Function. Wright Function.

3. Fractional Derivatives and Integrals

Grünwald-Letnikov Fractional Derivatives. Riemann-Liouville
Fractional Derivatives. Some Other Approaches. Geometric
and Physical Interpretation of Fractional Integration and
Fractional Differentiation. Sequential Fractional Derivatives.
Left and Right Fractional Derivatives. Properties of Fractional
Derivatives. Laplace Transforms of Fractional Derivatives.
Fourier Transforms of Fractional Derivatives. Mellin
Transforms of Fractional Derivatives.

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8. Numerical Solution of Fractional Differential Equations.

Initial Conditions: Which Problem to Solve?
Numerical Solution. Examples of Numerical
Solutions. The "Short-Memory" Principle in Initial
Value Problems for Fractional Differential
Equations. Matrix approach to discrete fractional
calculus. Numerical solution of nonlinear problems.

9. Survey of Applications

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4. Linear Fractional Differential Equations

Fractional Differential Equation of a General Form.
Existence and Uniqueness. Dependence of a Solution
on Initial Conditions. The Laplace Transform Method.
Standard Fractional Differential Equations. Sequential
Fractional Differential Equations.

5. Fractional Green's Function

Definition and Some Properties. One-Term Equation.
Two-Term Equation. Three-Term Equation. Four-Term
Equation. General Case: n-term Equation.

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Texts

- Podlubny I. *Fractional Differential Equations*.
San Diego: Academic Press; 1999.
- Oldham KB, Spanier J. *The Fractional Calculus*.
New York: Academic Press; 1974.
- Magin RL, *Fractional Calculus in Bioengineering*.
Begell House Publ., Connecticut, 2006.
- some other books and articles.

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Fractional Calculus

What's in the name?

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Fractional ...

- Constituting or comprising a part or fraction of a possible whole or entirety
- Date "fractional" was first used in popular English literature: sometime before 1762

(<http://www.websters-online-dictionary.org/definition/fractional>)

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...Calculus

- The branch of mathematics that is concerned with limits and with the differentiation and integration of functions
- Rhyming with "Calculus":
meticulous, miraculous, ridiculous,
fabulous, nebulous, populous, stimulus

(<http://www.websters-online-dictionary.org/definition/calculus>)

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